

Available online at [www.sciencedirect.com](http://www.sciencedirect.com)

Procedia Engineering 5 (2010) 1296–1299

**Procedia  
Engineering**[www.elsevier.com/locate/procedia](http://www.elsevier.com/locate/procedia)

Proc. Eurosensors XXIV, September 5-8, 2010, Linz, Austria

## The effect of thermoelastic damping on the total Q-factor of state-of-the-art MEMS gyroscopes with complex beam-like suspensions

T. Künzig<sup>1\*</sup>, M. Niessner<sup>1</sup>, G. Wachutka<sup>1</sup>, G. Schrag<sup>1</sup>, and H. Hammer<sup>2</sup><sup>1</sup>*Institute for Physics of Electrotechnology, Munich University of Technology, Munich, Germany*<sup>2</sup>*SensorDynamics AG, Graz-Lebring, Austria*

### Abstract

We present a comprehensive study of thermoelastic damping (TED) in state-of-the-art MEMS gyroscopes with beam-like suspensions under varying ambient temperatures. Three different modeling approaches have been implemented in a standard software environment (COMSOL Multiphysics) and compared with respect to accuracy, consistency and feasibility for fast design studies. By applying all three approaches, temperature-dependent Q-factors calculated for various designs of interest show congruent results proving the reliability of the simulations. Nevertheless, the theoretically determined values remain too high to explain the measured data by TED. As an additional result, it turned out that – under specific conditions – a simplified analytical model can be derived and applied successfully also to complex spring geometries, so as to produce a simple “rule of thumb” for first estimations and considerations in early stages of the design process.

© 2010 Published by Elsevier Ltd. Open access under [CC BY-NC-ND license](https://creativecommons.org/licenses/by-nc-nd/4.0/).*Keywords:* Thermoelastic damping; MEMS gyroscope; Quality factor;

### 1. Motivation

Predictive calculations and a profound understanding of loss mechanisms are a prerequisite for robust design of dynamically operated MEMS structures such as gyroscopes and MEMS resonators. In order to ensure high sensitivity, MEMS gyroscopes are encapsulated under low ambient pressure in order to reduce viscous damping to a minimum. Thus, other damping mechanisms, such as thermoelastic damping (TED), anchor loss or surface loss come to the fore [1]. Because of its high temperature dependence reported in [2], TED is of special interest to guarantee high quality factors over the required temperature range for automotive applications. In this work, we focus on thermoelastic damping in state-of-the-art gyroscope architectures with beam-like suspensions, which exhibit a strong temperature-dependent Q-factor under low ambient pressure conditions. Since TED cannot be measured independently from other damping mechanisms, three different FEM-based modeling approaches are pursued in order to assure the trustworthiness of the TED-related Q-factors.

\* Corresponding author. Tel.: +49-89-289-23108; fax: +49-89-289-23134.  
E-mail address: [kuenzig@tep.ei.tum.de](mailto:kuenzig@tep.ei.tum.de).

## 2. Theory and modeling approaches

The phenomenon of TED originates from the alternating strain in an oscillating structure, inducing temperature gradients due to an adiabatic volume change. Since the resulting heat flux is irreversible, the generated heat cannot be completely reclaimed by the mechanical oscillation and, thus, the resonating structure is damped significantly. This process is self-consistently described by the coupled thermo-mechanical system of equations shown in Fig. 1(a). In the following, three different approaches to solve the coupled system of equations are described in detail.

### Fully coupled approach

For the fully coupled approach, illustrated in Fig. 1(b), the full system of equations is implemented in a FEM-simulator including the complete bidirectional coupling. A complex thermo-mechanical modal analysis is carried out to extract the complex eigenvalue for the investigated oscillation mode. The Q-factor is obtained by calculating the ratio of imaginary to real part of the thermo-mechanical eigenvalue. This approach is described in detail in [3]. Since the full coupling is taken into account, this method offers the best accuracy and provides precise and quick results for simple device geometries. However, the memory consumption of the FEM simulator grows rapidly with increasing mesh points (4 DOF per mesh point), viz. with growing device complexity, and convergence problems may occur, if thermal and mechanical time scales differ widely; in this case, manual scaling of the dependent variables may be required.

### Weakly coupled approach

Another way to solve the system of equations is to neglect thermal expansion, which is true under the assumption of weak coupling. The mechanical domain is simulated separately and the coupling to the thermal domain is accomplished by applying an analytic formula, comprising the mechanical deflection and a modal decomposition of the temperature [4]. The Q-factor can then be obtained by calculating the ratio of energy stored in the resonator and dissipated energy per cycle (see Fig. 1(c)), resulting in

$$Q_{\text{TED}}^1 = \left( \frac{E\alpha^2 T_0}{\rho C_p} \right) \cdot \sum_n \frac{\omega \tau_n}{1 + (\omega \tau_n)^2} g_n \quad (1)$$

where  $E$  is the Young's modulus,  $\omega$  is the resonance frequency,  $\tau_n$  is the thermal decay constant of mode  $n$  and  $g_n$  is the modal weighting factor. The benefit of this approach is that the contribution of each single thermal mode to TED is calculated in equation (1). Hence, parts of the structure with high impact on TED can be identified easily, which facilitates the fast estimation of potential damping reduction significantly and distinguishes this approach as best

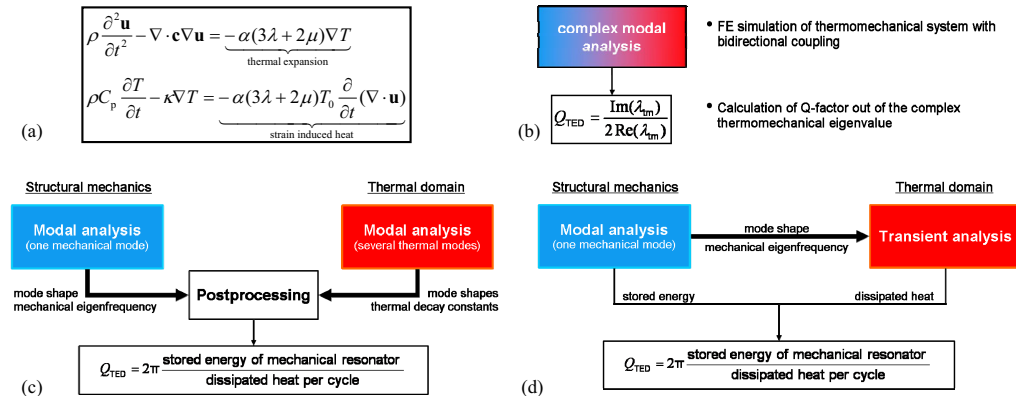


Fig. 1. (a) Fully coupled system of thermo-mechanical equations.

(c) Work flow of the weakly coupled approach.

(b) Work flow of the fully coupled approach.

(d) Work flow of the thermal energy method

$\rho$ : mass density,  $\mathbf{u}$ : displacement vector,  $\mathbf{c}$ : elasticity tensor,  $\alpha$ : thermal expansion coefficient,  $\lambda$  and  $\mu$ : Lamé parameters,  $T$ : temperature,  $C_p$ : specific heat capacity,  $\kappa$ : thermal conductivity and  $T_0$ : ambient temperature.

suited for design optimization. But for complex geometries a rising number of thermal modes has to be considered, which can enlarge the simulation times undesirably.

#### Thermal energy method

The third approach, described in [5], again neglects the thermal expansion and solves the mechanical equation independently (see Fig. 1(d)). The resulting deformation is used as an internal heat source for a transient thermal analysis and the so obtained temperature distribution is used to extract the dissipated heat per cycle and the Q-factor. This method provides the most intuitive and transparent way to calculate TED, however, the insight gained into fundamental processes is not as high as with the previous approach.

### 3. Results

In order to evaluate the consistency and accuracy of these methods and to avoid errors during implementation results for simple 2D test beams oscillating in the fundamental bending mode were compared (see Table 1). They agree very well and could be confirmed by the analytical model described in [6].

Subsequently, we calculated temperature dependent Q-factors to analyze the TED in the commercial rotational gyroscope, illustrated in Fig. 2(a). Since TED only occurs in strained regions, it is sufficient to concentrate on the spring and its anchors. Therefore, the movable mass of the gyroscope is emulated by an additional rectangular mass attached to the movable anchor, which is used to adapt the resonance frequency of the model to the drive mode frequency of the gyroscope. The displacement in the drive mode oscillation and the resulting heat flux density, responsible for TED, is visualized in Fig. 3(b) and (c). The obtained Q-factors (weakly coupled approach) of the gyroscope, displayed in Fig. 3, show the expected high temperature dependence. However, the magnitude of the simulated Q-factors is too high to have a remarkable effect on the total Q-factor, which is in the order of  $10^4$ . This leads us to the conclusion that the total Q-factor of the investigated gyroscope is not limited by TED, which means, the available measurements are not suited to verify the simulated TED-related Q-factors.

A detailed view on Fig. 2(c) shows that maximum heat flux density and hence major loss occurs in the regions close to the anchors, resembling the situation in a simple beam with temperature gradients along the beam width. Thus, in equation (1) of the weakly coupled approach, all dominant thermal processes are governed by the same

Table 1. Comparison of modeling approaches and analytical calculation for a 2D beam test structure.

Test structure: 2D beam (clamped-clamped)	$Q_{TED}$
Analytical model [6]	7 374
Fully coupled approach [3]	7 412
Weakly coupled approach [4]	7 445
Thermal energy method [5]	7 439

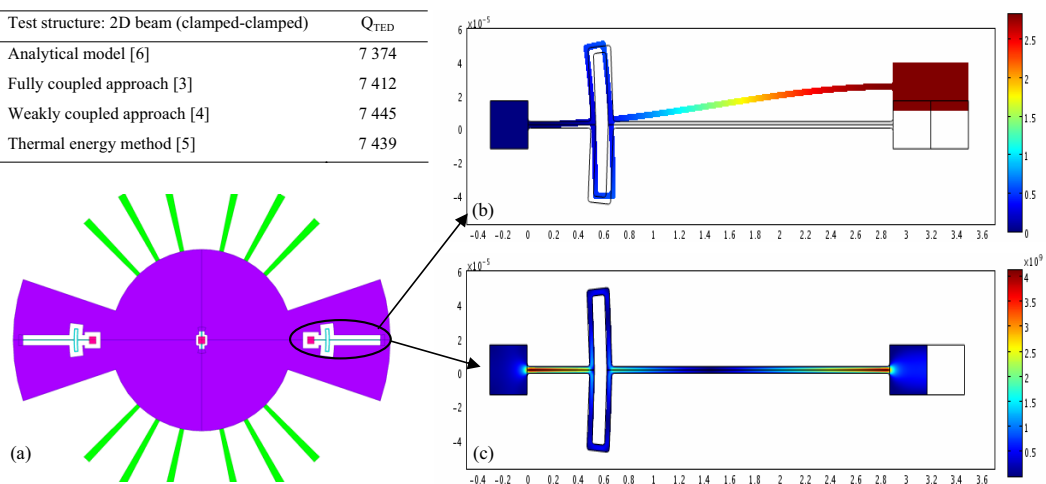


Fig. 2. (a) Geometry of an exemplary design of one of the rotational MEMS gyroscope variants under consideration: Anchors (red), springs (light blue), movable mass (dark blue). (b) Displacement of the gyroscope suspension in drive mode. (c) Heat flux density induced by deflection in drive mode effectuating the TED of the device.

decay constant  $\tau$  determined by the spring width  $b$ , and equation (1) may be simplified to eq. (2) below, which describes a model for rectangular beams developed by Zener [7-8]:

$$Q_{\text{TED}} = \left( \frac{\rho C_p}{E \alpha^2 T_0} \right) \frac{1 + (\omega \tau)^2}{\omega \tau} \quad (2)$$

A comparison of the simulated Q-factors with the Zener model, presented in Fig. 4, demonstrates that this model can – under certain assumptions – be used even for more complex beam-like structures to get a quick estimation of the magnitude of TED. For the investigated structure the error made by the Zener model approximation is about 12%, whereas the major reason for the difference is caused by additional loss in the anchor regions, which is not covered by this simplified model. The Zener model illustrates clearly the dominating factors for TED, namely the mechanical resonance frequency  $\omega$  and the thermal decay constant  $\tau$ , which itself depends on the spring width  $b$  [9].

For the considered gyroscope further simplification is feasible due to the low resonance frequency (kHz) compared to the time constants of the thermal processes (MHz) ( $\omega \ll 1/\tau$ ), which leads to

$$Q_{\text{TED}} \propto 1/(\omega b^2) \quad (3)$$

In conclusion, two parameters dominate the TED for gyroscopes with beam like suspensions: the resonance frequency  $\omega$  and the spring width  $b$ . Thus, if the trend of increasing resonance frequencies for automotive applications continues, increasing impact of TED can be expected in future gyroscope designs, since the spring widths cannot be made arbitrarily small.

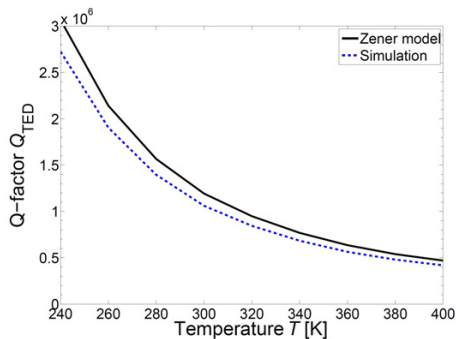


Fig. 3. Simulated temperature-dependent Q-factors of TED for the drive mode of the rotational gyroscope compared to the Zener model.

#### 4. Conclusions

We investigated the impact of TED on commercial gyroscope structures by applying different approaches to calculate this effect. All three implemented modeling approaches show congruent simulation results for 2D test-beams as well as for the investigated beam-like gyroscope structures, and the simulation results are confirmed by analytical calculations. For the investigated gyroscope designs, TED has only negligible impact on the overall damping, which is why these simulations cannot be confirmed by measurements. With the given assumptions, a quick estimation of TED in beam-like structures is enabled by the Zener model approximation. As critical design parameters affecting the TED the mechanical resonance frequency and the spring width have been identified.

#### References

- [1] Weinberg M, Candler R, Chandorkar S et al. *Energy Loss in MEMS Resonators and the Impact on Inertial and RF Devices*. Transducers 2009;688-95
- [2] Kim B, Hopcroft MA, Candler RN et al. *Temperature Dependence of Quality Factor in MEMS Resonators*. J Micro Sys 2008;17:755-66
- [3] COMSOL. *Thermoelastic Damping in a MEMS Resonator*. COMSOL Multiphysics Tutorial 2008
- [4] Duwel A, Candler RN, Kenny TW et al. *Engineering MEMS Resonators With Low Thermoelastic Damping*. J Micro Sys 2006;15:1437-45.
- [5] Hao Z, Xu Y, Durgam SK. *A thermal-energy method for calculating thermoelastic damping in micromechanical resonators*. J Sound and Vibration 2009;322:870-82
- [6] Lifschitz R, Roukes ML. *Thermoelastic damping in micro- and nanomechanical systems*. Phys Rev B 2000;61:5600-9
- [7] Zener C. *Internal Friction in Solids I. Theory of Internal Friction in Reeds*. Phys Rev 1937;52:230-35
- [8] Zener C. *Internal Friction in Solids II. General Theory of Thermoelastic Internal Friction*. Phys Rev 1938;53:90-9
- [9] Candler RN, Hopcroft M, Low CW et al. *Impact of Slot Location on Thermoelastic Dissipation in Micromechanical Resonators*. Transducers 2005;1:597-600